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The Eckhaus and the Benjamin-Feir Instability Near a Weakly Inverted Bifurcation

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NEAR A WEAKLY INVERTED BIFURCATION**

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Abstract

We investigate how the criteria for two prototype instabilities in one-dimensional pattern-forming systems, namely for the Eckhaus instability and for the Benjamin-Feir instability, change as one goes from a continuous bifurcation, to a spatially periodic or spatially and/or time periodic state, to the corresponding weakly inverted, i.e. hysteretic, cases. We also give the generalization to two-dimensional patterns in systems with anisotropy as they arise for example for hydrodynamic instabilities in nematic liquid crystals.

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1. Introduction

The question of which wavelengths are linearly stable in a system showing spatially periodic states has been of interest since about 25 years dating back to the classical work of Eckhaus¹. He investigated for a continuous bifurcation to a spatially periodic state how a given pattern with spatial variations in one direction (called one-dimensional pattern in the following) goes unstable upon applying a compressional or dilatational force. This instability, the Eckhaus instability, has since been studied theoretically for a number of pattern-forming systems showing a forward bifurcation, including Rayleigh-Bénard convection²⁻⁴ and the Taylor instability⁵. Experimentally this one-dimensional instability has been studied first for the onset of electroconvection in nematic liquid crystals⁶ and it has then been investigated in detail very carefully for Taylor vortex flow⁷.

For the case of a continuous bifurcation to a spatially and time periodic state in one dimension the question of a modulational instability was addressed first for a purely dispersive system (i.e. no dissipation) by Benjamin and Feir⁸, when they investigated how water waves go unstable. Their analysis has been generalized by Newell^{3,9-11} to include both dissipation and dispersion. Later on in ref.12 this analysis was further extended by considering the wavenumber band possible above onset of the instability and not only the critical wavenumber as in refs.3, 9-11. In addition, it has been shown¹³ that the Benjamin-Feir-Newell instability is equivalent to the phase diffusion coefficient in the Kuramoto-Sivashinsky equation becoming negative. Up to the present there seems to exist no clear-cut experimental observation of the Benjamin-Feir-Newell instability in one-dimensional patterns.

More recently there is an increasing interest in the spatial patterns formed near weakly inverted bifurcations (which show a small amount of hysteresis), especially when this bifurcation is oscillatory in nature¹⁴⁻²⁰. Motivated by these recent developments, which are partially triggered by experiments on thermal convection in binary fluid mixtures^{21,22}, we investigate here how the criteria for the Eckhaus instability and the Benjamin-Feir-Newell

instability change as one goes from a forward to a weakly inverted bifurcation.

2. The Eckhaus instability for a weakly inverted bifurcation to a spatially periodic state

The envelope equation for a weakly inverted bifurcation to a spatially periodic state reads

$$\dot{A} = \epsilon A + \gamma A_{xx} + \beta |A|^2 A - \delta |A|^4 A \quad (2.1)$$

where ϵ , γ , β and δ are real and where β and δ are assumed to be positive to guarantee that the bifurcation is inverted and saturates to quintic order. The diffusion coefficient γ will also be assumed to be positive.

The time-independent version of eq.(2.1) allows finite amplitude, plane wave solutions of the form

$$A_{st} = A_0 e^{ikz} \quad (2.2)$$

where

$$|A_0|_{1,2}^2 = \frac{\beta}{(2\delta)} \left(1 \pm \sqrt{1 + \frac{4(\epsilon - \gamma k^2)\delta}{\beta^2}} \right) \quad (2.3)$$

In eq.(2.3) the upper (+) sign in the bracket refers to the physical branch of solutions, which exists for a band of k values

$$\gamma k^2 < \frac{\beta^2}{4\delta} + \epsilon \quad (2.4)$$

whereas the lower (-) sign is associated with the unphysical branch of solutions.

To test for linear stability of the stationary solutions A_{st} (eq.(2.2)) against spatial modulations, we proceed along the lines of ref.2 and write

$$A = A_{st} + A_1(x, t) \quad (2.5)$$

We obtain upon linearization in A_1

$$\dot{A}_1 = \epsilon A_1 + \gamma A_{1xx} - A_1 |A_0|^2 (-2\beta + 3\delta |A_0|^2) - A_0^2 A_1^* e^{2ikz} (-\beta + 2\delta |A_0|^2) \quad (2.6)$$

This equation allows for solutions of the form

$$A_1(x, t) = A_{11}(t)e^{i(k+M)x} + A_{12}(t)e^{i(k-M)x} \quad (2.7)$$

Setting $A_{11}, A_{12} \propto e^{\lambda t}$ gives

$$\lambda_{1,2} = -(\gamma(k^2 + M^2) - \epsilon + |A_0|^2(-2\beta + 3\delta|A_0|^2)) \pm \sqrt{4\gamma^2 k^2 M^2 + |A_0|^4(-\beta + 2\delta|A_0|^2)^2} \quad (2.8)$$

$\lambda_{1,2}(M, k)$ has a relative maximum for

$$M^2 = k^2 - \frac{|A_0|^4(-\beta + 2\delta|A_0|^2)^2}{4\gamma^2 k^2} \quad (2.9)$$

and

$$\lambda = \epsilon - |A_0|^2(-2\beta + 3\delta|A_0|^2) + \frac{|A_0|^4}{4\gamma k^2}(-\beta + 2\delta|A_0|^2)^2 \quad (2.10)$$

In deriving eq. (2.10), the positive sign was chosen in eq. (2.8) since it corresponds to the larger growth rate. It is easily checked from eqs. (2.1), (2.9) and (2.10) that for $\delta = 0$ and β negative the classical result for a forward bifurcation^{1,2} (which reads

$$k^2 < \frac{\epsilon}{3\gamma} \quad (2.11)$$

in the present notation) is obtained. Inserting eq.(2.3) into eqs.(2.9), (2.10) we find that the finite amplitude solutions (eqs.(2.2, 2.3) with positive sign) of eq.(2.1) are Eckhaus stable, i.e. stable against the growth of sidebands, provided

$$k^2 < \frac{\epsilon}{2\gamma} + \frac{3\beta^2}{32\gamma\delta} \left(1 + \sqrt{1 + \frac{32\epsilon\delta}{9\beta^2}} \right) \quad (2.12)$$

Inequality (2.12) represents the stability domain for stable finite amplitude plane wave solutions of eq.(2.1) that are stable against the Eckhaus sideband instability. Larger values of the wavevector will lead to an instability. Comparison of inequalities (2.12) and (2.4) shows that the Eckhaus stable range is smaller than the existence range of nonlinear plane wave solutions of the type (2.2), (2.3). In analogy with the experimental observations^{6,7}

and with theoretical investigations²³ for a forward bifurcation one can expect the gain or loss of roll or vortex pairs to occur for the Eckhaus instability. This expectation is based on the fact that one has the same mechanism (a sideband instability) for both cases, namely for the spatially periodic states formed for a forward and for a weakly inverted bifurcation.

3. The analog of the Benjamin-Feir-Newell instability

For a weakly inverted bifurcation to a spatially and time periodic state one has for spatial variations in one dimension the envelope equation^{16-20,24,25}

$$\dot{A} = \epsilon A + \gamma A_{xx} - \beta |A|^2 A - \delta |A|^4 A \quad (3.1)$$

where γ , β , and δ are complex and thus of the form $\alpha = \alpha_r + i\alpha_i$ and where $\beta_r < 0$ and $\delta_r > 0$ to guarantee that the bifurcation is inverted and saturates to quintic order. Furthermore we have discarded nonlinear gradient terms in writing down eq.(3.1).

Eq.(3.1) admits spatially homogeneous solutions of finite amplitude

$$A_h = A_0 e^{i\omega t} \quad (3.2)$$

where

$$|A_0|_{1,2}^2 = -\frac{\beta_r}{2\delta_r} \left(1 \pm \sqrt{1 + \frac{4\epsilon\delta_r}{\beta_r^2}} \right) \quad (3.3)$$

and

$$\omega = -|A_0|^2(\beta_i + \delta_i|A_0|^2) \quad (3.4)$$

and where the (+) sign in the bracket of eq.(3.3) corresponds to the physical branch, whereas the (-) sign is associated with the unphysical branch. In the spirit of refs.9-11 we investigate the stability of the spatially homogeneous solutions (eqs.(3.2)-(3.4)) with respect to space-dependent perturbations, that is we look for solutions of eq.(3.1) of the form

$$A(x, t) = A_h(t) + B(x, t) \quad (3.5)$$

Inserting eqs.(3.2) - (3.5) into (3.1) we obtain upon linearization in B

$$\dot{B} = \epsilon B + \gamma B_{xx} - B|A_0|^2(2\beta + 3\delta|A_0|^2) - A_0^2 B^* e^{2i\omega t}(\beta + 2\delta|A_0|^2) \quad (3.6)$$

Using the ansatz

$$B = B_1(t)e^{ikx+i\omega_1 t} + B_2(t)e^{-ikx+i\omega_2 t} \quad (3.7)$$

and $2\omega = \omega_1 + \omega_2$ one gets a closed system of equations for $B_1(t)$ and $B_2^*(t)$. Setting $B_1(t) \propto B_{10}e^{\lambda t}$ and $B_2(t) \propto B_{20}e^{\lambda t}$, we arrive at the following solvability condition for λ

$$\begin{aligned} \lambda_{1,2} = & -(\gamma_r k^2 - \epsilon + 2\beta_r|A_0|^2 + 3\delta_r|A_0|^4) \\ & \pm \sqrt{|A_0|^4((\beta_r + 2\delta_r|A_0|^2)^2 + (\beta_i + 2\delta_i|A_0|^2)^2) - (\gamma_i k^2 + \omega + |A_0|^2(2\beta_i + 3\delta_i|A_0|^2))^2} \end{aligned} \quad (3.8)$$

Upon replacing ϵ and ω in eq. (3.8) with the values given by eqs. (3.3) and (3.4), respectively, analysis of eq.(3.8) shows that the spatially homogeneous solution (3.2) - (3.4) is linearly unstable against spatially inhomogeneous perturbations of the type considered here if

$$\gamma_i(\beta_i + 2\delta_i|A_0|^2) + \gamma_r(\beta_r + 2\delta_r|A_0|^2) < 0 \quad (3.9)$$

Eq.(3.9) represents the analog of the Benjamin-Feir-Newell criterion for a weakly inverted bifurcation. For $\delta_r = \delta_i = 0$ the case of the forward bifurcation^{3,9-11} is easily recovered from eq.(3.9)

$$\gamma_i\beta_i + \gamma_r\beta_r < 0 \quad (3.10)$$

As for the case of the forward bifurcation⁹⁻¹¹, the terms in eq.(3.9) represent a competition between the 'cooperative' (dissipative and diffusive) tendencies to generate an ordered structure at $q = q_0$ (the wavevector of the underlying structure such as for example rolls or vortices) and the dispersive forces (dispersion and nonlinear refractive index), which try to focus initial nonlinear wavetrains into pulses.

To investigate the modulational instability of plane wave solutions with a finite wavenumber we proceed along the lines of ref.12. Eq.(3.1) has finite amplitude plane

wave solutions of finite amplitude of the form

$$A_p = A_0 e^{i(kx + \omega t)} \quad (3.11)$$

where

$$|A_0|_{1,2}^2 = -\frac{\beta_r}{2\delta_r} \left(1 \pm \sqrt{1 + \frac{4\delta_r(\epsilon - \gamma_r k^2)}{\beta_r^2}} \right) \quad (3.12)$$

and

$$\omega = -\gamma_i k^2 - |A_0|^2 (\beta_i + \delta_i |A_0|^2) \quad (3.13)$$

and where the (+) sign in the bracket of eq.(3.12) corresponds to the physical branch, whereas the (-) sign is associated with the unphysical branch. To investigate the linear stability of the finite amplitude plane wave solutions (eqs.(3.11)-(3.13)) with respect to space-dependent perturbations we look for solutions of eq.(3.1) of the form

$$A(x, t) = A_p(x, t) + B(x, t) \quad (3.14)$$

We find that, linearizing in B , structurally the same equation for B is obtained (eq.(3.6)) as for the case of the spatially homogeneous solutions, except that now $|A_0|^2$ and ω are given by eqs.(3.12),(3.13) instead of eqs.(3.3),(3.4).

It is possible to find a solution of the resulting equation for B of the form

$$B(x, t) = a(t) e^{i(k_1 x + \omega_1 t)} + b(t) e^{i(k_2 x + \omega_2 t)} \quad (3.15)$$

provided $k_1 + k_2 = 2k$ and $\omega_1 + \omega_2 = 2\omega$. Introducing the abbreviations

$$\alpha_r = -\epsilon + 2\beta_r |A_0|^2 + 3\delta_r |A_0|^4 \quad (3.16)$$

$$\alpha_i = 2\beta_i |A_0|^2 + 3\delta_i |A_0|^4 \quad (3.17)$$

$$\zeta_r = \beta_r + 2\delta_r |A_0|^2 \quad (3.18)$$

$$\zeta_i = \beta_i + 2\delta_i |A_0|^2 \quad (3.19)$$

we find for $a(t)$ and $b^*(t)$ the equations

$$\dot{a} = -((\gamma_r + i\gamma_i)k_1^2 + i\omega_1 + (\alpha_r + i\alpha_i))a - (\zeta_r + i\zeta_i)A_0^2 b^* \quad (3.20)$$

$$\dot{b}^* = -((\gamma_r - i\gamma_i)k_2^2 - i\omega_2 + (\alpha_r - i\alpha_i))b^* - (\zeta_r - i\zeta_i)A_0^{*2} a \quad (3.21)$$

Assuming the disturbances to be proportional to $e^{\lambda t}$, we find that λ has a negative real part provided that

$$-\frac{1}{2}(\Sigma_1 + \Sigma_2) - \alpha_r + N \cos\left(\frac{1}{2}\theta\right) < 0 \quad (3.22)$$

where

$$N = \left(\left(\frac{1}{4}(\Sigma_1 - \Sigma_2)^2 - \frac{1}{4}(\Gamma_1 + \Gamma_2)^2 + \Lambda^2 \right)^2 + \frac{1}{4}(\Gamma_1 + \Gamma_2)^2(\Sigma_1 - \Sigma_2)^2 \right)^{\frac{1}{4}} \quad (3.23)$$

and

$$\tan\theta = \frac{\frac{1}{2}(\Gamma_1 + \Gamma_2)(\Sigma_1 - \Sigma_2)}{\frac{1}{4}(\Sigma_1 - \Sigma_2)^2 - \frac{1}{4}(\Gamma_1 + \Gamma_2)^2 + \Lambda^2} \quad (3.24)$$

with

$$\Gamma_j = \alpha_i + \gamma_i k_j^2 + \omega_j \quad (3.25)$$

$$\Sigma_j = \gamma_r k_j^2 \quad (3.26)$$

$$\Lambda = |A_0|^2 |\zeta| \quad (3.27)$$

where $j = 1, 2$. Comparing eqs.(3.22)-(3.27) with eqs.(3.14) - (3.18) of ref.12, we see that there is a great similarity in structure. Further analysis of inequality (3.22) turns out to be rather involved for the case of a weakly inverted bifurcation and is left to future investigations. In closing this section we note, that eqs.(3.22) - (3.27) contain as a special case inequality (3.9), as one sees after some algebraic manipulations.

4. Generalization to anisotropic systems

So far we have looked at envelope equations containing only spatial variations in one spatial dimension. Now we generalize this to the case of spatial variations in two dimensions

for systems which have an intrinsic (such as anisotropic liquids: nematic liquid crystals etc.) or extrinsic (e.g. due to an external force) anisotropy as it occurs for example for hydrodynamic instabilities in magnetic liquids in an external magnetic field.

In this case the envelope equation (2.1) for a weakly inverted bifurcation to a spatially periodic state is replaced by

$$\dot{A} = \epsilon A + \gamma_{11} A_{xx} + \gamma_{12} A_{xy} + \gamma_{22} A_{yy} + \beta |A|^2 A - \delta |A|^4 A \quad (4.1)$$

with γ_{11} , γ_{12} , and γ_{22} real.

An envelope equation for a forward bifurcation to a spatially periodic state has been derived for electroconvection in nematic liquid crystals²⁶. As for the case of the forward bifurcation²⁶ one can diagonalize the terms containing second derivatives of the envelope in eq.(4.1) by rotating the coordinate system and by rescaling the coordinates. Then the derivative terms take the form of a two-dimensional Laplacian Δ_2 and the resulting envelope equation for a weakly inverted bifurcation to a spatially periodic state reads

$$\dot{A} = \epsilon A + \tilde{\gamma} \Delta_2 A + \beta |A|^2 A - \delta |A|^4 A \quad (4.2)$$

It is easy to show that the most unstable perturbations correspond to perturbations along the plane wave and therefore all the steps we have outlined in section 2 for the Eckhaus instability in one dimension apply. We therefore obtain for the band of Eckhaus-stable wavevectors

$$k_\xi^2 + k_\eta^2 < \frac{\epsilon}{2\tilde{\gamma}} + \frac{3\beta^2}{32\tilde{\gamma}\delta} \left(1 + \sqrt{1 + \frac{32\epsilon\delta}{9\beta^2}} \right) \quad (4.3)$$

where ξ and η in inequality (4.3) denote the coordinates in which the spatial derivative terms in eq.(4.1) assume the structure of the two-dimensional Laplacian.

For a weakly inverted bifurcation to a spatially and time periodic state in an anisotropic system, eq.(3.1) is replaced by

$$\dot{A} = \epsilon A + \gamma_{11} A_{xx} + \gamma_{12} A_{xy} + \gamma_{22} A_{yy} - \beta |A|^2 A - \delta |A|^4 A \quad (4.4)$$

where γ_{11} , γ_{12} , and γ_{22} are complex and of the form $\gamma_{kl} = \gamma_{kl}^r + i\gamma_{kl}^i$. For the envelope equation (4.4) we can investigate the analog of the Benjamin-Feir-Newell instability along the same lines as for the one-dimensional case in section 3. We find that the analysis goes through and that inequality (3.9) is replaced by the requirement that the (2x2) matrix Σ_{kl}

$$\Sigma_{kl} = \gamma_{kl}^i(\beta_i + 2\delta_i|A_0|^2) + \gamma_{kl}^r(\beta_r + 2\delta_r|A_0|^2) \quad (4.5)$$

must be positive to guarantee linear stability of finite amplitude plane wave solutions against space-dependent perturbations.

This analysis might turn out to be useful in the near future, since an inverted bifurcation to traveling waves has been found recently experimentally for the onset of electroconvection in nematic liquid crystals²⁷. In addition it has been predicted a few years ago theoretically²⁸, that the onset of thermal convection in nematics can also arise as a weakly inverted oscillatory instability. We note that this calculation has been done for homeotropic alignment of the director. The preferred direction in the plane is provided by the external destabilizing magnetic field, which is applied in a direction parallel to the nematic layer. Thus the planar horizontal symmetry is not broken by the orientation of the director due to surface treatment as in the planar configuration studied in electroconvection, but broken externally by the direction of the external in-plane magnetic field.

For the case of a forward oscillatory bifurcation, the condition that the matrix in eq.(4.5) must be positive for stability is replaced by the requirement that the (2x2) matrix $\tilde{\Sigma}_{kl}$

$$\tilde{\Sigma}_{kl} = \gamma_{kl}^i\beta_i + \gamma_{kl}^r\beta_r \quad (4.6)$$

must be positive for stability, which is a direct application of a previous general theoretical result by Newell¹⁰ to a real physical system, namely electrohydrodynamic instabilities in nematic liquid crystals. Envelope equations for a forward bifurcation to traveling waves have been discussed very recently²⁹ in an analysis of experimental results on electroconvection in nematics³⁰, where a transition from traveling to standing waves could be achieved

by modulation.

5. Summary and perspective

In this note we have discussed the analogues of the Eckhaus and the Benjamin-Feir-Newell instability for a weakly inverted bifurcation. In addition, we have generalized these criteria to the case of anisotropic liquids such as nematic liquid crystals.

From an experimental point of view the next step could be the investigation of the Benjamin-Feir-Newell instability for traveling waves in thermal convection in binary fluid mixtures, which are well documented for the upper (stable) branch of the hysteresis loop. This will allow a test of the criterion given here, provided one chooses a value for the separation ratio such that the size of the hysteresis loop is sufficiently small and thus an envelope equation is applicable. To test the criterion given for the Eckhaus instability for a weakly inverted bifurcation to a spatially periodic pattern one could also study binary convective mixtures. For such mixtures in a porous medium a regime of separation ratios has been found over which the bifurcation is expected to be inverted^{31,14}. For the case of anisotropic fluids such as nematic liquid crystals, it will be most interesting to see under what circumstances the bifurcation to the stationary branch is actually forward or inverted. In the latter case, which has been observed for the onset of electroconvection³², one could test the criterion presented here for the Eckhaus instability near a weakly inverted bifurcation to a stationary pattern.

Note added: After submitting this paper for publication, we became aware of ref.33. This paper contains in eq.(1.5) implicitly our criterion (eq.(3.9)) for the analogue of the Benjamin-Feir-Newell instability for a weakly inverted bifurcation. Due to the scaled form of eq.(1.1) in ref.33, however, it is not possible to go through the tricritical point from the weakly inverted case to a forward bifurcation. In our description this can be done straightforwardly (compare section 3).

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References

- 1 W. Eckhaus, *Studies in nonlinear stability theory* Springer, Berlin 1965
- 2 A.C. Newell and J.A. Whitehead, *J.Fluid Mech.* **38**, 279 (1969)
- 3 A.C. Newell and J.A. Whitehead, p.284 in *Proceedings of the IUTAM Symposium on instability in continuous systems*, H. Leipholz, Ed., Springer, N.Y. 1971
- 4 L. Kramer and W. Zimmermann, *Physica D - Nonlinear Phenomena* **16**, 221 (1985)
- 5 S. Kogelman and R.C. DiPrima, *Phys.Fl.* **13**, 1 (1970)
- 6 M. Lowe and J.P. Gollub, *Phys.Rev.Lett.* **55**, 2575 (1986)
- 7 G. Ahlers, D.S. Cannell, M.A. Dominguez-Lerma, and R. Heinrichs, *Physica D - Nonlinear Phenomena* **23**, 202 (1986)
- 8 T.B. Benjamin and J.E. Feir, *J.Fluid Mech.* **27**, 417 (1966)
- 9 A.C. Newell, p.157, in *Lectures in Appl.Math.* M. Kac., Ed. **15** (1974)
- 10 A.C. Newell, p.244 in *Proceedings of an international workshop on synergetics*, H. Haken, Ed., Springer, N.Y. 1979
- 11 A.C. Newell, *Solitons in Mathematics and Physics*, Society for Industrial and Applied Mathematics, Philadelphia, 1985
- 12 J.T. Stuart and R.C. DiPrima, *Proc.Roy.Soc.* **A362**, 27 (1978)
- 13 Y. Kuramoto and T. Tsuzuki, *Prog.Theor.Phys.* **54**, 687 (1975)
- 14 V. Steinberg and H.R. Brand, *Phys.Rev.* **A30**, 3366 (1984)
- 15 H.R. Brand and B.J.A. Zielinska, *Phys.Rev.Lett.* **57**, 3167 (1986)
- 16 R.J. Deissler and H.R. Brand, *Phys.Lett.* **A130**, 293 (1988)
- 17 O. Thual and S. Fauve, *J.Phys.(Paris)* **49**, 1829 (1988)
- 18 H.R. Brand and R.J. Deissler, *Phys.Rev.Lett.* **63**, 2801 (1989)
- 19 W. van Saarloos and P.C. Hohenberg, *Phys.Rev.Lett.* **64**, 749 (1990)
- 20 R.J. Deissler and H.R. Brand, *Phys.Lett.* **A146**, 252 (1990)
- 21 P. Kolodner, D. Bensimon, and C.M. Surko, *Phys.Rev.Lett.* **60**, 1723 (1988)
- 22 J.J. Niemela, G. Ahlers, and D.S. Cannell, *Phys.Rev.Lett.* **64**, 1365 (1990)

- 23 L. Kramer, H.R. Schober, and W. Zimmermann, *Physica D - Nonlinear Phenomena* **31**, 212 (1988)
- 24 H.R. Brand, P.S. Lomdahl and A.C. Newell, *Phys.Lett.* **A118**, 67 (1986)
- 25 H.R. Brand, P.S. Lomdahl and A.C. Newell, *Physica D - Nonlinear Phenomena* **23**, 345 (1986)
- 26 L. Kramer, E. Bodenschatz, W. Pesch, W. Thom, and W. Zimmermann, *Liq.Cryst.* **53**, 699 (1989)
- 27 I. Rehberg, S. Rasenat, M. de la Torre, W. Schöpf, F. Hörner, G.Ahlers, and H.R. Brand, *Phys.Rev.Lett.* **67**, 596 (1991)
- 28 E. Dubois-Violette and M. Gabay, *J.Phys.(Paris)* **43**, 1305 (1982), and E. Dubois-Violette, private communication
- 29 D. Walgraef, *J.Stat.Phys.* in print (1991)
- 30 I. Rehberg, S. Rasenat, J. Fineberg, M. de la Torre, and V. Steinberg, *Phys.Rev.Lett.* **61**, 2449 (1988)
- 31 H.R. Brand and V. Steinberg, *Phys.Lett.* **A93**, 333 (1983)
- 32 M. Takata, S. Kai, and K. Hirakawa, *Technology Reports of Kyushu University (Fukuoka, Japan)* **51**, 539 (1978) (in Japanese), and S. Kai, private communication
- 33 B.A. Malomed and A.A. Nepomnyashchy, *Phys.Rev.* **A42**, 6009 (1990)

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13. ABSTRACT (Maximum 200 words) We investigate how the criteria for two prototype instabilities in one-dimensional pattern-forming systems, namely for the Eckhaus instability and for the Benjamin-Feir instability, change as one goes from a continuous bifurcation, to a spatially periodic or spatially and/or time periodic state, to the corresponding weakly inverted, i.e. hysteretic, cases. We also give the generalization to two-dimensional patterns in systems with anisotropy as they arise for example for hydrodynamic instabilities in nematic liquid crystals.				
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